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Micromechanical modeling of the effect of particle size difference in dual phase steels

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Abstract

Dual phase (DP) steels having a microstructure consisting of martensite islands, referred to as particles, dispersed in a ferrite matrix have received a great deal of attention due to their useful combination of high strength, high work hardening rate and ductility, all of which are favorable properties for forming processes. The martensite particles display two distinct deformation mechanisms, depending on their size. Small particles are reported in the literature to undergo no measurable plastic deformation and thus can be described as rigid particles dispersed in a matrix of ferrite. On the other hand, large particles reportedly experience a small degree of plastic deformation, which has a significant influence on the mechanism of deformation of such materials. Although most micromechanical models assume a uniform particle size, a distribution of sizes in DP-steels is a more realistic assumption. In this work, a micromechanical model is developed to capture the effect of particle size differences on the mechanical behavior of DP-steels. It is shown that the difference becomes most significant when the ratio of the small to large particle size is approximately 1/2. At low volume fractions of martensite, the effect of a distribution of particle sizes is negligible, but at intermediate and high volume fractions of martensite the interaction due to the size difference becomes quite important. The model displays the intrinsic ability of capturing the steep rise in the strain-hardening rate observed in DP-steels. The model also successfully predicts the mechanisms involved in the deformation process in the DP-steels in agreement with experimental observations reported in the literature.

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1. Introduction

It is well established that low carbon multiphase (MP) steels developed in the past decades offer impressive mechanical properties such as high work hardening rate and good ductility which also have the advantage of reduced cost, superior formability, and excellent surface finish over other high strength low alloy (HSLA) steels. The advantages of dual phase (DP)-steels were first reported by Rashid (1976), who

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has shown significant increase in strength compared to the commercial plain carbon steels, but are inferior in terms of ductility and formability. A comprehensive review of the various aspects of DP-steels is reported in a previous work by Al-Abbasi and Nemes (submitted).

DP-steels are reported to undergo three distinct deformation processes (Rashid and Cprek, 1978; Bag et al., 1999; Byun and Kim, 1993; Tomota, 1987). In the first stage both the ferrite matrix and the martensite particles deform elastically. In the second stage the ferrite phase deforms plastically while the martensite phase continues to deform elastically. In the third stage both the ferrite and martensite phases deform plastically.

Shen et al. (1986) have reported that the distribution of the strains between the martensite and ferrite phases as well among the different grains of each phase to be inhomogeneous. The ferrite phase was observed to deform immediately and at a rapid rate followed by the delayed deformation of martensite. Rashid and Cprek (1978) have also shown that the martensite phase deforms after excessive straining of the ferrite matrix due to load being transferred to the martensite phase through the martensite–ferrite interface.

2. Micromechanical modeling

The unique mechanical properties of the MP-steels are attributable to their microstructure, which can be considered on several levels, all of which influence the final behavior of the material. Modeling on the steel phase level is proven to be the least expensive computationally and most rational, as the constituents can be considered homogeneous while from atomic to grain levels of the structure the properties are not realistically represented by an isotropic continuum. Micromechanical models are used to understand the local mechanics and mechanisms governing the macroscopic elastic–plastic deformation of heterogeneous solids. They provide overall behavior from known properties of the individual constituents and their detailed interaction unlike the macromechanical approach where the heterogeneous structure behavior has to be known to predict the aggregate behavior using a computational model. The micromechanical modeling of cells, an approach for predicting the macroscopic mechanical behavior of two material systems in which one material is dispersed with spherical inhomogeneities, is well established. The prominent feature of this approach is the transition from a medium with a periodic microstructure to an equivalent homogeneous continuum, which effectively represents the material. The three basic features of the micromechanical approach are: the geometric definition of a representative volume element (RVE) which possesses the essential features of the microstructure, the constitutive description of the mechanical behavior of each phase and the interaction between them and a homogenization procedure based on the RVE to get the macroscopic material behavior.

2.1. Representative volume element (RVE)

In a previous work by Al-Abbasi and Nemes (submitted), it was shown that the micromechanical approach based on the stacked hexagonal array (SHA) model could well represent this type of material as it displays intrinsic ability to capture the expected stress–strain behavior with increasing V_m of the second phase and in terms of deformation fields of the constituents. Other models based on the plane strain idealization were shown to be unable to represent or capture the mechanical behavior of DP-steels, especially at high V_m .

In this work the RVE will be based on the same axisymmetric model but with two different particle sizes. The size difference in DP-steels, as seen in Fig. 1, is a more realistic assumption that can be made to model this kind of material instead of assuming a single size of particle. The idealization used in this model is shown in Fig. 2 where periodic hard particles of two sizes are dispersed in the softer matrix. Due to

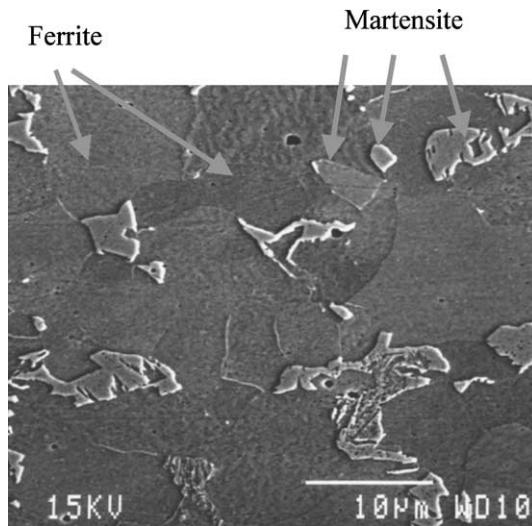


Fig. 1. Typical microstructure of dual-phase steel showing the variation of the particle sizes.

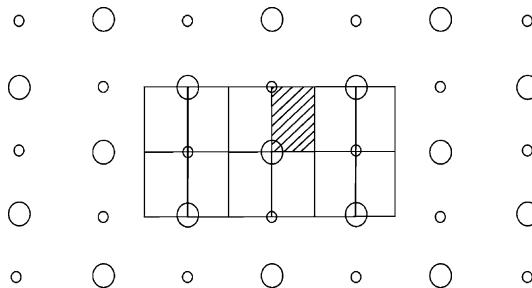


Fig. 2. Axisymmetric two-particle model RVE idealization, showing the homogeneous dispersion of small and large particles.

symmetry only the dashed area is modeled. The small and large particles are chosen to be staggered with respect to each other to keep the material homogeneous.

2.2. The constitutive behavior of each material phase

The constitutive behaviors of bainite, martensite, pearlite and approximated behavior of ferrite have been reported in the literature (Bourell and Rizk, 1983; Ishikawa et al., 2000). In the micromechanical model, the constitutive behavior of the constituents will only be required to investigate the aggregate behavior, which is thus far achievable. The interaction of phases (interface boundaries) will be ignored, as it is considerably small, on the order of few atomic sizes, compared to the phases being modeled. In addition, the interface boundary between the martensite and ferrite in a DP-steel, produced by quenching from the intercritical annealing temperatures to room temperature, is considered fully coherent. Thus a perfectly continuous boundary between the ferrite and martensite has been used in the micromechanical model.

Each phase is considered to be an elastic–plastic solid and it is assumed that the strain ‘rate’ can be additively decomposed into elastic and plastic components,

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (1)$$

where Hooke's law describes the elastic component. The plastic strain rate is given as

$$\begin{aligned} \dot{\epsilon}_{ij}^p &= 0 \quad f < 0 \\ \dot{\epsilon}_{ij}^p &= \frac{3}{2} \frac{\dot{\epsilon}^p}{\sigma} \sigma'_{ij} \quad f = 0 \end{aligned} \quad (2)$$

where the deviatoric stresses $\sigma'_{ij} = \sigma_{ij} - (1/3)\sigma_{kk}$ and the equivalent stress, σ , and the equivalent strain rate, $\dot{\epsilon}^p$, are defined as

$$\begin{aligned} \sigma &= \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} \\ \dot{\epsilon}^p &= \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p} \end{aligned} \quad (3)$$

The von Mises yield condition is assumed:

$$f = \sigma - \bar{\sigma} \quad (4)$$

where $\bar{\sigma}$ is a function of the equivalent plastic strain and is taken to describe the isotropic hardening.

The hardening behavior of the two phases is taken from the experimental results obtained by Davies (1978) and expressed by the following:

$$\begin{aligned} \bar{\sigma}_f &= K_f (\epsilon_0 + \dot{\epsilon}_f^p)^{n_f} \\ \bar{\sigma}_m &= K_m (\epsilon_0 + \dot{\epsilon}_m^p)^{n_m} \end{aligned} \quad (5)$$

where the subscripts f and m denote ferrite and martensite, respectively, and ϵ_0 is taken to be equal to 0.002 in this work, K_m and K_f are taken to be 2409 and 597 MPa, respectively and n_m and n_f are 0.07 and 0.31, respectively. The stress vs. plastic strain for each phase is shown in Fig. 3.

To better illustrate the difference in uniform elongation between the two phases, the behavior under uniaxial tension stress is considered. Under these conditions, the equivalent stress reduces to the true uniaxial stress, σ_{true} , and the equivalent plastic strain is equal to the plastic strain along the direction of loading or true strain, ϵ_{true} . The corresponding engineering quantities are related to the true quantities as follows:

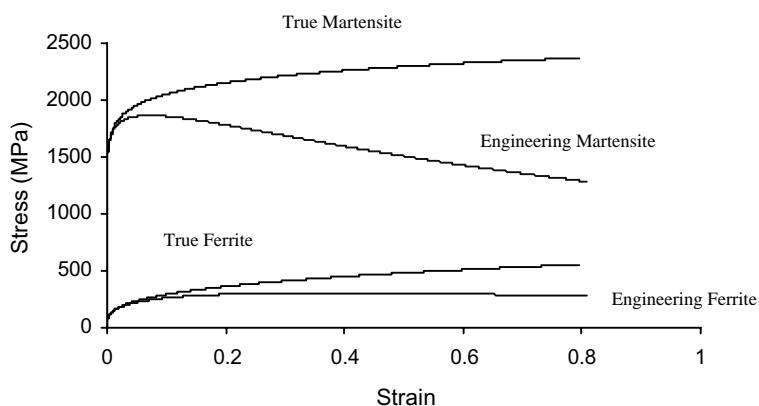


Fig. 3. Behavior of the martensite and ferrite phases shown as true stress vs. true strain and engineering stress vs. engineering strain for uniaxial tensile stress conditions.

$$\begin{aligned}\varepsilon_{\text{eng}} &= \exp(\varepsilon_{\text{true}}) - 1 \\ \sigma_{\text{eng}} &= \sigma_{\text{true}} / (1 + \varepsilon_{\text{eng}})\end{aligned}\quad (6)$$

The engineering stress vs. engineering strain response for uniaxial tension loading is also shown in Fig. 3, where the difference in uniform strain (strain at maximum engineering stress) for the two phases is quite apparent.¹

2.3. Homogenization method

The macroscopic stress components are computed as the volume average of the microscopic components according to the following equations:

$$S_{ij} = \frac{1}{V} \int_V \sigma_{ij} dV \quad (7)$$

$$E_{ij} = \frac{1}{V} \int_V \varepsilon_{ij} dV \quad (8)$$

where S_{ij} and E_{ij} are the macroscopic average component of stresses and strains over the microscopic volume of the micromechanical model. The macromechanical behavior of the aggregate is, therefore, approximated by the volume average of the micromechanical behavior.

Many early works in micromechanical modeling focused on voids within a solid matrix. McClintock (1968) considered the evolution of a single cylindrical void in an infinite matrix subjected to axisymmetric loading at the remote boundary. Rice and Tracey (1969) followed by investigating the response of an isolated spherical void in an infinite medium. Both authors considered a rigid perfectly plastic material. Gurson (1977) proposed approximate yield criteria for ductile porous media using a micromechanical approach. He considered a representative cell containing a spherical void and proposed a plastic potential function for porous materials which has the advantage of accounting for or characterizing damage and fracture in ductile materials. Since Gurson's model is based on the assumption that the deformation mode of the matrix material surrounding the void is homogeneous, it can predict material softening behavior due to nucleation and growth of voids but fails to predict the transition from homogeneous to inhomogeneous deformation where coalescence of voids take place. Tvergaard (1981) used Gurson's yield criteria and introduced the micromechanical modeling of cells based on a random distribution of particles that can be idealized by considering a regular three-dimensional array of hexagonal cylinders of a matrix material, each containing a spherical void or particle. The problem was further simplified by modeling axisymmetric geometry, where Tvergaard assumed that an infinite series of stacked circular cylinders containing spherical particles is a good approximation for the three-dimensional stacked hexagonal array. Symmetry arguments are then used to limit the RVE to 1/4 of the axisymmetric cell. Different investigators have used the Gurson-Tvergaard model to investigate the mechanical behavior of a wide variety of materials. Socrate and Boyce (2000) and Al-Abbasi and Nemes (submitted) give comprehensive reviews of this work.

In real void containing materials, there is a distribution of void sizes resulting from different sizes of inclusions at which voids nucleate. Although the behavior of voids is quite different than that of elastic-plastic particles, the effect of a distribution of sizes raises interesting possibilities for the micromechanical modeling of DP-steels. In a study of the micromechanics of coalescence, Faleskog and Shih (1997)

¹ It should be noted that no fracture criteria has been employed so the response shown cannot predict the limiting strain value of the phase under tensile loading.

introduced a representative material volume containing several large voids and a population of microvoids present from the very beginning using a plane strain model. They modeled all voids as discrete entities and have shown that a local zone of high stress concentration emanates from the large void and spreads across the material raising the stresses at the nearby microvoids. As a result, the microvoids grew unstably. Tvergaard (1996) investigated the effect of void size difference on growth and cavitation instabilities. His analysis was based on an axisymmetric unit cell model, which allows for the representation of a number of spherical voids. Tvergaard has shown that for a range of rather low stress triaxiality, that the relative growth rates of the two voids vary with initial void volume fraction and for very high stress triaxialities interaction effects become important if the initial void volume fraction is sufficiently low and predicted that only one of the voids grew large. In another work Tvergaard (1998) investigated the interaction of very small voids with large voids to determine whether or not local stress increases induced by the large void result in cavitation instability at the tiny void. He has shown that for overall stress levels as large as those reached ahead of a blunting crack tip cavitation instability does not form at the small voids due to the interaction with the large void but the results show that localization of plastic flow in the unit cell plays an important role. Orsini and Zikry (2001) in a study of void growth and interaction in crystalline materials have used a micromechanical model with discrete voids at seven different positions to study the interrelated physical mechanisms that can result in ductile fractures. They used a rate dependent constitutive formulation and allowed the plane strain micromodel to neck by not restricting the side of the model to be straight. Although the ligament between the voids neck in the microlevel, this cannot be a realistic representation of the material behavior since continuity would not be preserved.

As in a material containing voids, the assumption of different particle sizes in a heterogeneous material is more realistic than one with a single particle size. Socrate et al. (2001) in a study of multiple crazing in high impact, toughened, polystyrene have reported that particle diameters in such materials are in the range 1–4 μm with an average size around 2 μm . The particles smaller than 0.8 μm , which corresponds to 1/4 the particle size, do not initiate crazes, unlike larger particles. They have also reported that the most effective particle size for toughening is in the range 1–2 μm , which is between the average and one-half the average particle size. In this work, a micromechanical model for the DP-steels, which allows for the investigation of the effect of particle size difference, is developed. The point of interest is whether or not the size difference in DP-steels has any effect on the predicted response of such materials.

3. Finite element modeling

Finite element analysis has been used to carry out the homogenization procedure. The analyses considered were limited to 2D, axisymmetric case, as the plane strain models are proven to overpredict the strain hardening of DP-steels as shown by Al-Abbas and Nemes (submitted), and to keep computational time reasonable. The commercial code ABAQUS was used to perform the analysis. Each phase, namely, martensite and ferrite, is considered to be an elastic–plastic solid as described by Eqs. (1)–(5), with $E = 200$ GPa and $\nu = 0.3$.

The mesh used in this model is made of two identical upper and lower parts. The mesh around each hard particle is chosen to be identical to avoid any effect of meshing on the analysis. Referring to Fig. 4b, the volume fraction is computed as $[2(a^3 + b^3)]/[3(L^2H)]$. Symmetry boundary conditions are used for sides S1 and S2, while side S3 has a uniform displacement in the x_1 direction and side S4 has uniform displacement in x_2 direction. The aggregate strains are computed as

$$E_{11} = \ln(u_1(L, x_2)/L) \quad (9)$$

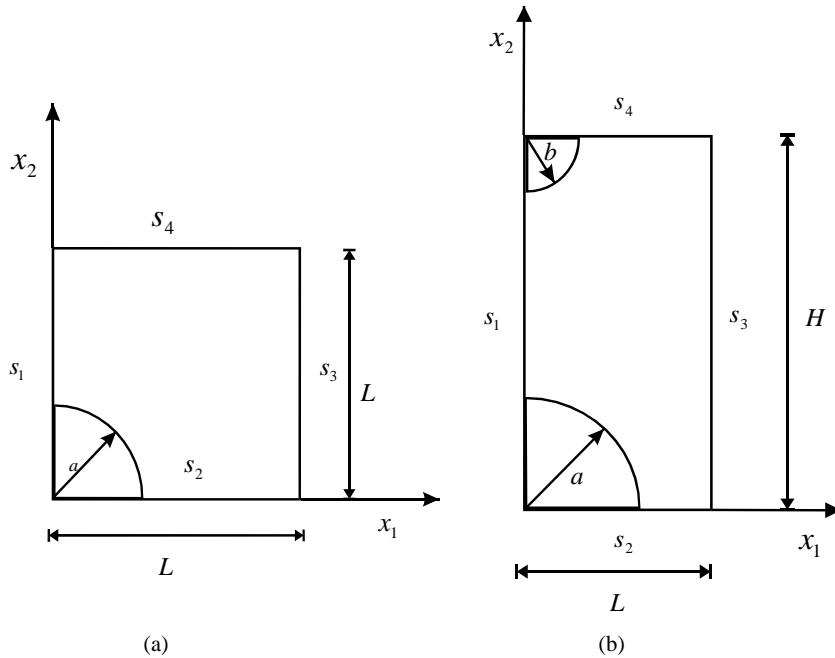


Fig. 4. Dimensions of the micromechanical models: (a) single particle model (b) two particle model.

$$E_{22} = \ln(u_2(x_1, H)/H) \quad (10)$$

The engineering normal stress in the x_2 direction is computed from the resultant force divided by the original area, from which using (6), the Cauchy stress component is computed. For the axisymmetric micromodels the state of stress is uniaxial, such that the S_{22} and E_{22} components of stress and strain are equal to the corresponding equivalent quantities.

In the analysis carried out in this work, two meshes are considered. One has a single martensite particle and equal side dimensions (mesh1) and the other one has two martensite particles and unequal side dimensions ($H = 2L$), which facilitate the investigation into the effect of size difference on the mechanical properties of DP-steels. Both meshes are illustrated in Fig. 4. The evolution of the plastic strain and stress-strain response for mesh1 and mesh2 with equal particle sizes ($a = b$; $R = b/a = 1$) at the same V_m are identical. Mesh2 displays the expected obvious symmetry when the two particles are of equal size (see Fig. 4) as depicted in the equivalent plastic strain evolution shown in Fig. 5 for $V_m = 13\%$.

The response of the aggregate at different particle size ratios ($R = b/a$) at $V_m = 6.8\%$, 17% and 21.5% is shown in Fig. 6a–c respectively. It can clearly be seen that the contribution of the particle size on the predicted response of the model is very small at low V_m , while it is considerably more important at intermediate and high V_m . The plot of the predicted tensile strength, vs. R is shown in Fig. 7 for the case of $V_m = 17\%$. The figure shows that the strength of the aggregate increases from $R = 1$ to $3/4$ and reaches a maximum value at $R = 1/2$. Reducing the value of R further reduces the strength but the reduction is quite small. The other cases ($V_m = 6.8\%$ and 21.5%) follow the same trend, but with different values for the ultimate strength.

The predicted response of the model for low values of R , i.e. $R = 1/16$ would be expected to decline to values similar to the response of the model at $R = 1$ since the smaller particle becomes negligible. However, as indicated in Figs. 6 and 7, this is not the case, which led the authors to investigate the effect of the dimensions H and L on the response of the model with unequal parts. The response of the model at

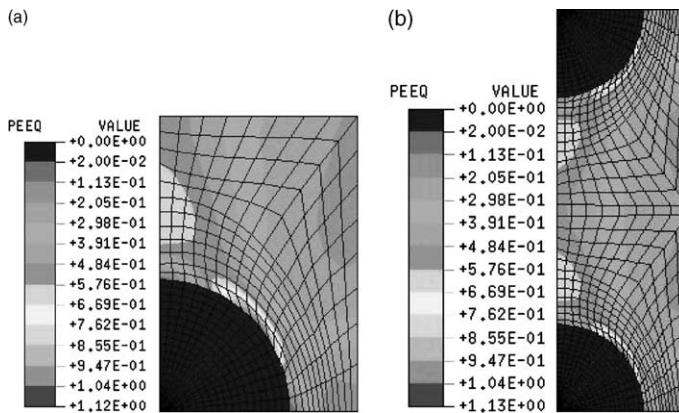


Fig. 5. The evolution of the equivalent plastic strain for the single particle model and the two particle model at particle size ratio, $R = 1$.

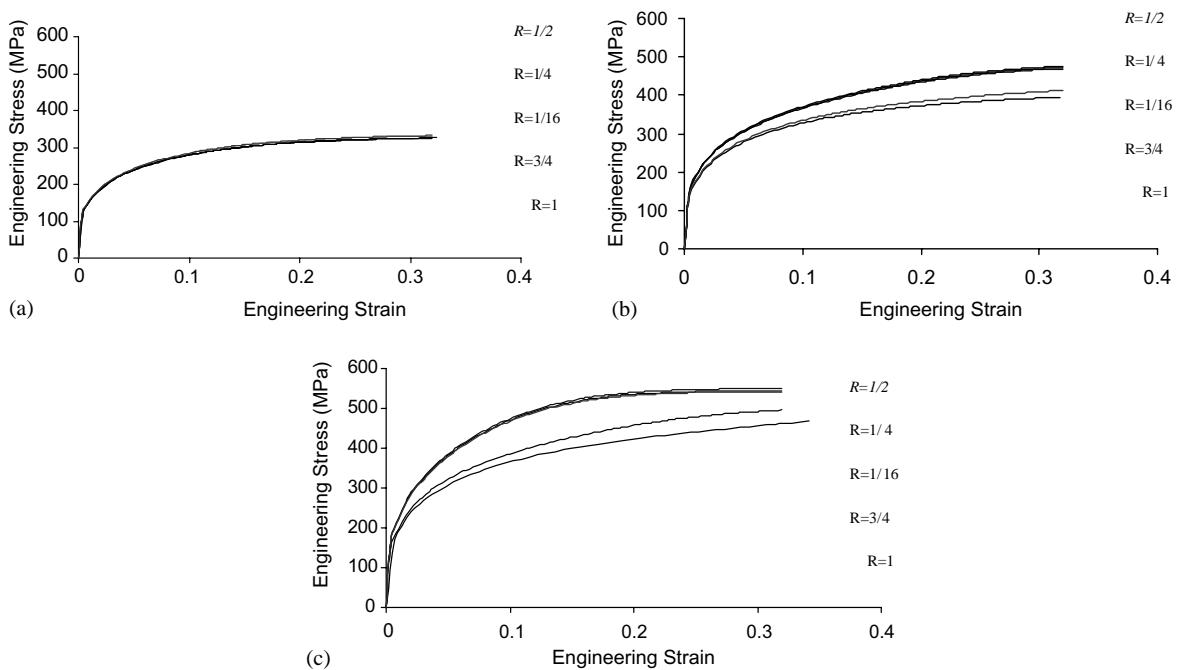


Fig. 6. The predicted response of the two particle model at three different volume fractions of the harder phase (a) 6.8% (b) 17% (c) 21.5% for particle size ratios of $R = 1, 3/4, 1/2, 1/4, 1/16$.

$R = 1/16$ is compared to the same model at $R = 0$ (only one particle) and found to be identical, but different from results using mesh1 for the same volume fraction, which clearly indicates that the dimensions of the cell (H and L) have an influence on the response of the model. Similar effects have been observed by Pardoen and Hutchinson (2000) in a study of a model for void growth and coalescence.

The effect that particle size distribution has on the response of DP-steels can also be seen by examination of contours of equivalent plastic strain. Fig. 8 shows the distribution of plastic strain for several cases of combinations of volume fraction and R value. At low V_m (Fig. 8a) the martensite particles do not plastically

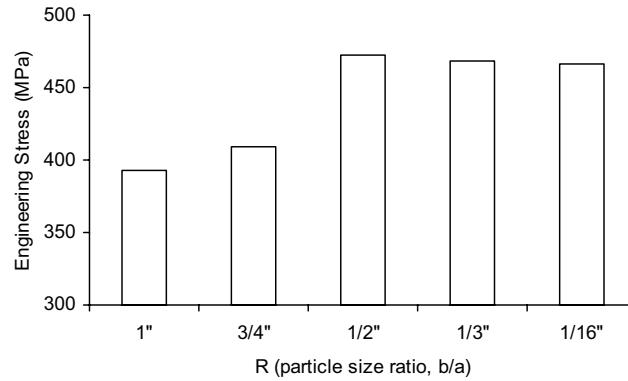


Fig. 7. The predicted tensile strength of the aggregate at the particle size ratios, $R = 1, 3/4, 1/2, 1/3$ and $1/16$ for $V_m = 17\%$.

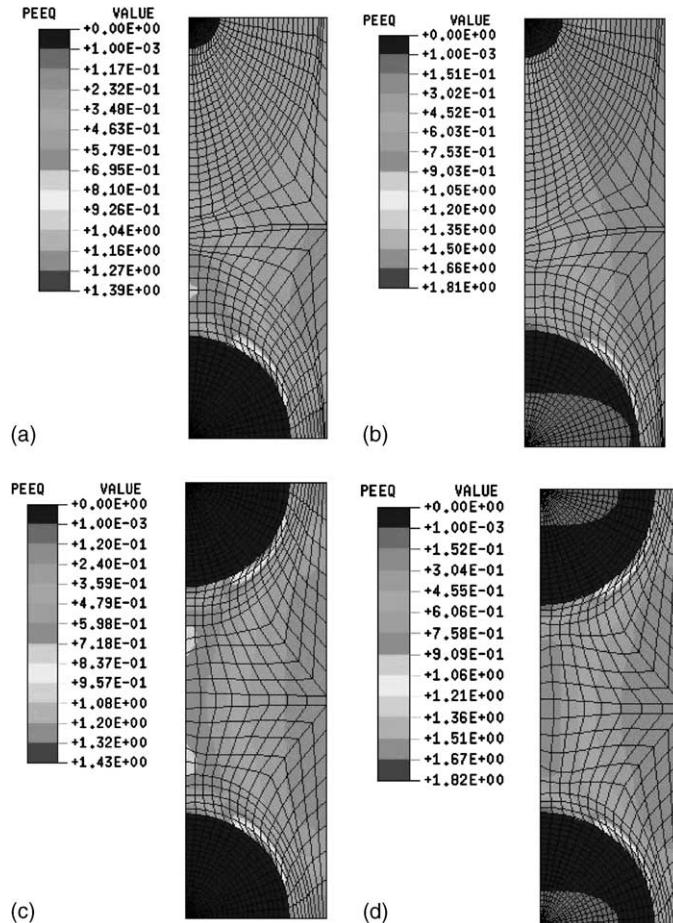


Fig. 8. Contours of equivalent plastic strain for nominal strain of 30%: (a) $R = 1/4, V_m = 9\%$; (b) $R = 1/4, V_m = 13\%$; (c) $R = 1, V_m = 17\%$; (d) $R = 1, V_m = 22\%$.

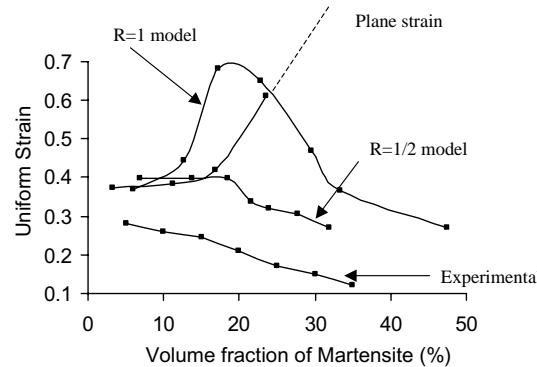


Fig. 9. Experimental results (Davies, 1978) of uniform strain vs. volume fraction of martensite compared to results using three modeling assumptions.

deform, which is consistent with results found for $R = 1$, and also explains the negligible effect of particle size distribution seen in the stress vs. strain response of Fig. 6a. However, at higher volume fractions of martensite (Fig. 8b), the distribution of plastic strain in the martensite is heterogeneous, with the large particle exhibiting plastic strain as high as 15% and the smaller particle remaining elastic. By comparing Fig. 8b and c, it can also be seen that a heterogeneous distribution of particle sizes results in plastic deformation in the martensite occurring at lower volume fractions. These differences are manifested in the differences of the stress vs. strain response apparent at medium and high martensite volume fractions seen in Fig. 6.

Quantitative comparison of the results predicted by the model to experimental data are difficult to perform because even small differences in chemistry or grain size between the materials used for the constituent properties here and that used in available experiments on DP-steel will have significant influence on the behavior. Nevertheless, certain aspects of the behavior of DP-steels, which is attributed to their inherent nature can be examined. One of these characteristics is the decrease in uniform strain observed with increasing volume fraction of martensite (Rashid and Cprek, 1978), which is an important trade-off to obtain increased strength. Experimental results of uniform strain as a function of volume fraction of martensite from Davies (1978) is shown in Fig. 9 along with predicted results using three different modeling assumptions. Clearly, only the model containing the heterogeneous particle size distribution captures the same trend as the experimental results, although quantitative differences are noted.

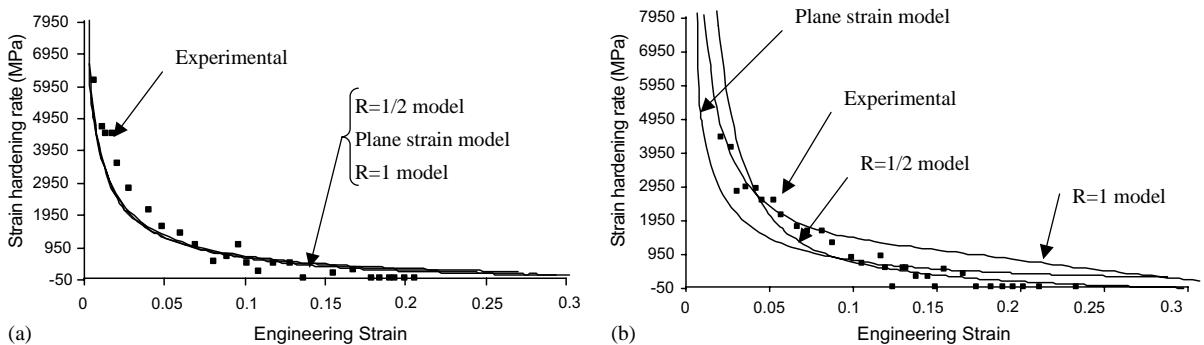


Fig. 10. Experimental results of strain hardening rate from Shen et al. (1986) compared to three modeling assumptions (a) low V_m (b) high V_m .

Another characteristic of DP-steels attributed to their heterogeneous nature is their high UTS to yield ratio or high strain hardening rate, which is particularly attractive for many forming operations. Fig. 10 shows the strain-hardening rate vs. engineering strain using data from Shen et al. (1986) at two martensite volume fractions compared to results from different modeling assumptions. At $V_m = 10\text{--}13.8\%$ (low V_m) the closest trend is displayed by the model with $R = 1/2$ even though the difference is not significant. At $V_m = 30\text{--}33\%$ (high V_m) the model with $R = 1$ shows better agreement with the experiment up to 3% strain but from 4% strain onwards the model with $R = 1/2$ shows close agreement with the experimental result while the other two models deviate significantly from the experimental results. The comparisons in Figs. 9 and 10 clearly show that the model with $R = 1/2$ is able to better capture the strain hardening rate from the early stages of straining to the onset of instability than the model with $R = 1$ and the plane strain model.

4. Conclusion

Different particle size ratios have been examined to investigate the size ratio effect on the micromechanical model to evaluate its importance in prediction of the response of the material. The ratios considered were $R = 1, 3/4, 1/2, 1/4, 1/16$ and 0. As mentioned earlier, $R = 1$ presents the symmetric case where the results are identical to the single particle case as in mesh1 and thus no effect of particle size interaction is noticed. As one of the particles is reduced in size at a constant volume fraction of martensite the effect of the difference in particle size becomes more noticeable and reaches its maximum effect at particle size ratio $R = 1/2$. Reducing the smaller particle size further to $R = 1/4$ and $1/16$ was physically expected to result in the decline of the stress-strain curve to reach $R = 1$ response due to the little effect of the very small particle. This trend was noticed but with a very limited decrease in stress values, a fact which is attributed to the difference in cell dimensions, as noted previously. Changing the RVE cell geometry ratio will bring in the effect of particle distribution in the matrix, which was not covered in this work, but has the same effect on the cell model when the cell dimension L is reduced and H is kept constant.

Further examination of the results show that at low V_m and particle size difference of $R = 1/2$ there is no measurable difference in the response compared to the model with $R = 1$. At high V_m the effect is clear as shown in Fig. 6. In addition, the difference comes about only when the larger martensite particle undergoes plastic deformation and this occurs at the larger particle only as shown in Fig. 8b. The onset of plastic deformation of the larger martensite particle in the $R = 1/2$ model occurs at lower volume fractions of martensite than the particles in the $R = 1$ model. This takes place due to the fact that the ferrite phase surrounding the larger particle is very constrained compared to the ferrite in the latter case at the same V_m , which forces the larger martensite particle to undergo earlier plastic deformation. It is worthwhile mentioning that Shen et al. (1986) reported martensite deformation levels for various combinations of V_m and %C in the steel. Increasing the V_m with constant %C in the steel causes dilution of the carbon in the martensite particles, which reduces its strength, but this is reported to be significant only in volume fractions above 30%. Since the purpose of this work is to investigate the effect of particle size distribution on the mechanical behavior of DP-steels, this effect is not considered, although it is believed to have an influence on the overall behavior of the material.

Different straining models have been used in the past by various authors in analyzing the strength of DP-steels. Supporters of simple rule of mixtures (Davies, 1978) or Mileiko's theory (Ramos et al., 1997) of mixtures of two ductile phases, considered the equal strain model or Voigt estimate suitable for DP-steels. Others who have followed Ashby's theory of particle strengthening supported equal stress models or Reuss estimates considered that martensite can only be strained after necking. Both estimates have been shown by Hill (1963) to be the upper and lower bounds, respectively. It was shown by Al-Abbas and Nemes (submitted) that, a large discrepancy in the upper and lower bounds exists for typical DP-steels. Authors who investigated intermediate models like Speich and Miller (1979) and Szewczyk and Gurland (1982)

considered the difference between the strain in the martensite and ferrite but held the strain ratio constant throughout the entire tensile process. A comprehensive review of the above is given by Szewczyk and Gurland (1982) and Korzekwa et al. (1980). Nevertheless, none of the models above were verified experimentally. Lei and Shen (1981) have shown that during cold rolling of DP-steel 1020, the deformation process undergoes a transition from unequal strain during initial stages to a constant strain ratio at the final stages of deformation. Shen et al. (1986) have also shown using a scanning electron microscopy equipped with a straining stage that at low V_m only the ferrite matrix deforms with no measurable strain occurring in the martensite particles. At high V_m , however, they have shown that shearing of the interface between the martensite and ferrite occurs extending the strain into the martensite islands after the ferrite matrix is excessively strained, which is in agreement with Rashid and Cprek (1978). They have also shown that the distribution of the strains between the ferrite and martensite phases, as well as among the different grains of each phase was observed to be inhomogeneous. Furthermore, they have shown that during initial stages of straining the slip process occurred in the larger ferrite grains only. The smaller ferrite grains withstood the lower strains and that after necking the weaker martensite islands began to deform under the action of shear strain concentrated in the neighboring ferrite grains in agreement with Rashid and Cprek (1978).

From the analysis above it is noticed that only the larger particle experiences plastic deformation while the smaller particle undergoes no measurable plastic deformation. The difference in the strain distribution among the martensite particles can be attributed to the size effect since the plastic deformation in the martensite particles takes place due to the strain or load being transferred from the ferrite matrix to the martensite particles. Since the small particles with small interface surface would take less strain than the larger particle sizes, the larger particles deform faster than the smaller ones and the small ones do not experience any plastic deformation.

In agreement with the above experimental observations, the model with two particle sizes shows two different mechanisms of deformation occurring in the martensite particles. The larger particle neighboring the highly strained ferrite matrix, which Shen et al. (1986) considered as weak martensite particles, undergo plastic deformation at intermediate and high V_m , while the smaller particle experiences no plastic deformation. In other words, the larger particle shows a duplex deformation mechanism while the smaller martensite particle shows only a particle strengthening mechanism.

In this work a micromechanical model for the DP-steels consisting of martensite particles of two different sizes dispersed in a ferrite matrix has been developed. The model captures the effect of the realistic assumption of different particle sizes being present and distinguishes two mechanisms of deformations taking place side by side in the process of tensile straining. Due to the complex nature of strain hardening and due to different mechanisms being present in the deformation process, previous authors failed to develop a mathematical model that can completely describe the deformation process in DP-steels. In addition to being successful in capturing the trend of increasing strength and decreasing uniform strain with increasing volume fraction of martensite, this model can capture the effect of different particle sizes and predict the existence of two mechanisms of deformation during the deformation process. In doing so, the model also captures the onset of plastic instability implied by the uniform strain and the interesting steep increase in the strain hardening rate, which distinguishes DP-steels from other types of steels.

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